

The super-heat-kernel expansion and the renormalization of the pion–nucleon interaction^{*}

H. Neufeld^{1,2}

¹ CERN, CH-1211 Geneva 23, Switzerland

² Institut für Theoretische Physik, Universität Wien, Boltzmannngasse 5, A-1090 Vienna, Austria

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Abstract. A recently proposed super-heat-kernel technique is applied to $SU(2)_L \times SU(2)_R$ heavy baryon chiral perturbation theory. A previous result for the one-loop divergences of the pion–nucleon system to $\mathcal{O}(p^3)$ is confirmed, giving at the same time an impressive demonstration of the efficiency of the new method. The cumbersome and tedious calculations of the conventional approach are now reduced to a few simple algebraic manipulations. The present computational scheme is not restricted to chiral perturbation theory, but can easily be applied or extended to any (in general non-renormalizable) theory with boson–fermion interactions.

1 Introduction

The modern treatment of the pion–nucleon system as an effective field theory of the standard model was pioneered by Gasser, Sainio, Švarc [1] and Krause [2] who formulated the “relativistic” version of baryon chiral perturbation theory. It was then shown by Jenkins and Manohar [3] that the methods of heavy quark effective theory [4] allow for a systematic low-energy expansion of baryonic Green functions in complete analogy to the meson sector. The latter approach is usually called heavy baryon chiral perturbation theory.

Applications of this effective field theory beyond the tree level require the knowledge of the divergences generated by one-loop graphs. For the pion–nucleon interaction in the heavy mass expansion, the full list of one-loop divergences to $\mathcal{O}(p^3)$ has been worked out by Ecker [5]. This analysis was then extended to the three-flavour case by Müller and Meißner [6]. In these papers, the bosonic loop and the mixed loop (boson and fermion lines in the loop) were treated separately. This required a cumbersome investigation of the singular behaviour of products of propagators, because the mixed loop does not have the form of a determinant, like the purely bosonic or fermionic loops.

To overcome these difficulties, we have recently developed [7] a method where bosons and fermions are treated on an equal footing. Employing the notions of supermatrices, superdeterminants and supertraces [8, 9], we have constructed a super-heat-kernel representation for the one-loop functional of a boson-fermion system. In this approach, the determination of the one-loop divergences is reduced to simple matrix manipulations, in complete anal-

ogy to the familiar heat-kernel expansion technique for bosonic or fermionic loops.

The present paper is organized as follows: in Sect. 2 I briefly review the super-heat-kernel method. In contrast to the Euclidean space formulation used in [7], the presentation in this work refers to Minkowski space throughout. In Sect. 3 the super-heat-kernel formalism is applied to a rather general class of scalar–heavy fermion interactions (including, of course, heavy baryon chiral perturbation theory). The one-loop divergences to second order in the fermion fields are given explicitly. These results are then specialized to the two-flavour version of heavy baryon chiral perturbation theory in Sect. 4. My conclusions, together with an outlook to possible extensions of the present work, are summarized in Sect. 5. Several momentum-space integrals are collected in the Appendix.

2 Super-heat-kernel

Let us consider a general action

$$S[\varphi, \psi, \bar{\psi}] = \int d^d x \mathcal{L}(\varphi, \psi, \bar{\psi}) \quad (2.1)$$

for n_B real scalar fields φ_i and n_F spin 1/2 fields ψ_a . Anticipating the later use of dimensional regularization, I am starting in d -dimensional Minkowski space. To construct the generating functional Z of Green functions, these fields are coupled to external sources j_i ($i = 1, \dots, n_B$), $\rho_a, \bar{\rho}_a$ ($a = 1, \dots, n_F$),

$$\begin{aligned} Z[j, \rho, \bar{\rho}] &= e^{iW[j, \rho, \bar{\rho}]} \\ &= \int [d\varphi d\psi d\bar{\psi}] e^{i(S[\varphi, \psi, \bar{\psi}] + j^T \varphi + \bar{\psi} \rho + \bar{\rho} \psi)}, \end{aligned} \quad (2.2)$$

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where $W[j, \rho, \bar{\rho}]$ is the generating functional of connected Green functions. I have used the notation

$$j^T \varphi + \bar{\psi} \rho + \bar{\rho} \psi := \int d^d x (j_i \varphi_i + \bar{\psi}_a \rho_a + \bar{\rho}_a \psi_a). \quad (2.3)$$

The normalization of the functional integral is determined by the condition $Z[0, 0, 0] = 1$. The solutions of the classical equations of motion

$$\frac{\delta S}{\delta \varphi_i} + j_i = 0, \quad \frac{\delta S}{\delta \bar{\psi}_a} + \rho_a = 0, \quad \frac{\delta S}{\delta \bar{\rho}_a} - \bar{\rho}_a = 0 \quad (2.4)$$

are denoted by $\varphi_{\text{cl}}, \psi_{\text{cl}}$. They are uniquely determined functionals of the external sources. With fluctuation fields ξ, η defined by

$$\varphi_i = \varphi_{\text{cl},i} + \xi_i, \quad \psi_a = \psi_{\text{cl},a} + \eta_a, \quad (2.5)$$

the integrand in (2.2) is expanded in terms of $\xi, \eta, \bar{\eta}$. The resulting loop expansion of the generating functional

$$W = W_{L=0} + W_{L=1} + \dots$$

starts with the classical action in the presence of external sources:

$$W_{L=0} = S[\varphi_{\text{cl}}, \psi_{\text{cl}}, \bar{\psi}_{\text{cl}}] + j^T \varphi_{\text{cl}} + \bar{\psi}_{\text{cl}} \rho + \bar{\rho} \psi_{\text{cl}}. \quad (2.6)$$

The one-loop term $W_{L=1}$ is given by a Gaussian functional integral

$$e^{iW_{L=1}} = \int [d\xi d\eta d\bar{\eta}] e^{iS^{(2)}[\varphi_{\text{cl}}, \psi_{\text{cl}}, \bar{\psi}_{\text{cl}}; \xi, \eta, \bar{\eta}]}, \quad (2.7)$$

where

$$S^{(2)}[\varphi_{\text{cl}}, \psi_{\text{cl}}, \bar{\psi}_{\text{cl}}; \xi, \eta, \bar{\eta}] = \int d^d x \mathcal{L}^{(2)}(\varphi_{\text{cl}}, \psi_{\text{cl}}, \bar{\psi}_{\text{cl}}; \xi, \eta, \bar{\eta}) \quad (2.8)$$

is quadratic in the fluctuation variables. Employing the notation introduced in (2.3), $S^{(2)}$ takes the general form

$$S^{(2)} = \frac{1}{2} \xi^T A \xi + \bar{\eta} B \eta + \xi^T \bar{\Gamma} \eta + \bar{\eta} \Gamma \xi = \frac{1}{2} \left(\xi^T A \xi + \bar{\eta} B \eta - \eta^T B^T \bar{\eta}^T + \xi^T \bar{\Gamma} \eta - \eta^T \bar{\Gamma}^T \xi + \bar{\eta} \Gamma \xi - \xi^T \Gamma^T \bar{\eta}^T \right), \quad (2.9)$$

where $A, B, \Gamma, \bar{\Gamma}$ are operators in the respective spaces; $A = A^T$ and B are bosonic differential operators, whereas Γ and $\bar{\Gamma}$ are fermionic (Grassmann) operators. They all depend on the classical solutions $\varphi_{\text{cl}}, \psi_{\text{cl}}$.

The standard procedure for the evaluation of (2.7) is to integrate first over the fermion fields $\eta, \bar{\eta}$ to yield the bosonic functional integral

$$e^{iW_{L=1}} = \det B \int [d\xi] e^{\frac{i}{2} \xi^T (A - \bar{\Gamma} B^{-1} \Gamma + \Gamma^T B^{-1} \bar{\Gamma}^T) \xi}.$$

This leads to the familiar result

$$\begin{aligned} W_{L=1} &= \frac{i}{2} \left[\ln \det \left(A - \bar{\Gamma} B^{-1} \Gamma + \Gamma^T B^{-1} \bar{\Gamma}^T \right) \right. \\ &\quad \left. - \ln \det A_0 \right] - i (\ln \det B - \ln \det B_0) \\ &= \frac{i}{2} \text{Tr} \ln \frac{A}{A_0} - i \text{Tr} \ln \frac{B}{B_0} \\ &\quad + \frac{i}{2} \text{Tr} \ln \left(1 - A^{-1} \bar{\Gamma} B^{-1} \Gamma + A^{-1} \Gamma^T B^{-1} \bar{\Gamma}^T \right) \\ &= \frac{i}{2} \text{Tr} \ln \frac{A}{A_0} - i \text{Tr} \ln \frac{B}{B_0} - \sum_{n=1}^{\infty} \frac{i}{2n} \\ &\quad \times \text{Tr} \left(A^{-1} \bar{\Gamma} B^{-1} \Gamma - A^{-1} \Gamma^T B^{-1} \bar{\Gamma}^T \right)^n, \end{aligned} \quad (2.10)$$

$$A_0 := A|_{j=\rho=\bar{\rho}=0}, \quad B_0 := B|_{j=\rho=\bar{\rho}=0}.$$

Recalling that A^{-1}, B^{-1} are the scalar and fermion matrix propagators in the presence of external sources, the one-loop functional $W_{L=1}$ is seen to be a sum of the bosonic one-loop functional $\frac{i}{2} \text{Tr} \ln(A/A_0)$, the fermion-loop functional $-i \text{Tr} \ln(B/B_0)$ and a mixed one-loop functional where scalar and fermion propagators alternate. In order to determine the ultraviolet divergences that occur in the mixed term in (2.10), the calculational inconveniences mentioned in Sect. 1 are encountered.

These problems can be circumvented [7] by reorganizing the three parts of $W_{L=1}$ into a more compact form, using the notion of supermatrices, supertraces, etc. (see for instance [8,9]). Combining the bosonic and fermionic fluctuation variables in a multicomponent field

$$\lambda = \begin{bmatrix} \xi \\ \eta \\ \bar{\eta}^T \end{bmatrix}, \quad (2.11)$$

$S^{(2)}$ in (2.9) can be written as

$$S^{(2)} = \frac{1}{2} \lambda^T K \lambda. \quad (2.12)$$

The explicit form of the supermatrix operator K follows immediately from the second line in (2.9):

$$K = \begin{bmatrix} A & \bar{\Gamma} & -\Gamma^T \\ -\bar{\Gamma}^T & 0 & -B^T \\ \Gamma & B & 0 \end{bmatrix}. \quad (2.13)$$

The one-loop functional of connected Green functions can now be written in compact form [7] in terms of a supertrace

$$W_{L=1} = \frac{i}{2} \text{Str} \ln \frac{K}{K_0}. \quad (2.14)$$

With the notation

$$\text{Str} O = \int d^d x \text{str} \langle x | O | x \rangle$$

I distinguish supertraces with and without space-time integration.

For actual calculations, the form of the supermatrix operator K defined in (2.13) is not the most convenient one. Applying a similarity transformation to K , the generating functional can also be written as

$$W_{L=1} = \frac{i}{2} \text{Str} \ln \frac{K'}{K'_0} \quad (2.15)$$

with

$$K' = \begin{bmatrix} A & \sqrt{\mu} \bar{\Gamma} & -\sqrt{\mu} \Gamma^T \\ \sqrt{\mu} \Gamma & \mu B & 0 \\ \sqrt{\mu} \bar{\Gamma}^T & 0 & \mu B^T \end{bmatrix}. \quad (2.16)$$

The arbitrary mass parameter μ introduced in (2.16) guarantees equal dimensions for all entries in K' ($[K'] = [A] = 2$). Although this quantity does not, of course, appear in any final result, it turns out to be quite helpful for the inspection of expressions at intermediate stages of calculations.

In the proper-time formulation, the one-loop functional assumes the form

$$\begin{aligned} W_{L=1} &= -\frac{i}{2} \int_0^\infty \frac{d\tau}{\tau} \text{Str} \left(e^{i\tau K'} - e^{i\tau K'_0} \right) \\ &= -\frac{i}{2} \int_0^\infty \frac{d\tau}{\tau} \int d^d x \text{str} \left\langle x \left| e^{i\tau K'} - e^{i\tau K'_0} \right| x \right\rangle, \end{aligned} \quad (2.17)$$

which is just the desired super-heat-kernel representation. Note that the convergence of the integral at the upper end ($\tau \rightarrow \infty$) is guaranteed by the small imaginary parts present in the bosonic and fermionic differential operators A and B , which are ensuring at the same time the usual Feynman boundary conditions. (For a free theory $A = -\square - M^2 + i\epsilon$, $B = i \not{\partial} - m + i\epsilon$.) On the other hand, the behaviour of the integral at the lower end exhibits the divergence structure of the theory under investigation.

As long as we are only interested in those parts of the one-loop functional that are at most bilinear in fermion fields, the supermatrix K' can be reduced to the simpler form

$$K'' = \begin{bmatrix} A & \sqrt{2\mu} \bar{\Gamma} \\ \sqrt{2\mu} \Gamma & \mu B \end{bmatrix}, \quad (2.18)$$

such that the one-loop functional reads

$$W_{L=1} = \frac{i}{2} \text{Str} \ln \frac{K''}{K''_0} - \frac{i}{2} \text{Tr} \ln \frac{B}{B_0} + \dots \quad (2.19)$$

The terms omitted are at least quartic in the fermion fields.

3 Scalars interacting with heavy fermions

In the case of chiral perturbation theory with heavy baryons, the fluctuation action (2.9) generated by the lowest order meson–baryon Lagrangian ($\mathcal{O}(p^2)$ in the mesonic and $\mathcal{O}(p)$ in the baryonic part) has the general form

$$\begin{aligned} S^{(2)} &= -\frac{1}{2} \xi^T (D_\mu D^\mu + Y) \xi + \bar{\eta} (\alpha + \beta_\mu D^\mu) \xi \\ &\quad + \xi^T (\bar{\delta} - \bar{\beta}_\mu D^\mu) \eta + \bar{\eta} i v_\mu D^\mu \eta, \end{aligned} \quad (3.1)$$

where

$$\begin{aligned} D_\mu &= \partial_\mu + X_\mu, \quad \mathcal{D}_\mu = \partial_\mu + f_\mu, \quad \bar{\delta} = \bar{\alpha} - \overline{\mathcal{D}_\mu \beta^\mu}, \\ v^2 &= 1, \quad v \cdot \beta = 0. \end{aligned} \quad (3.2)$$

X_μ , Y and f_μ are bosonic (matrix-) fields, whereas α and β_μ are fermionic objects. The form of $\bar{\delta}$ in (3.2) is required by the reality of (3.1). v_μ is the usual velocity vector introduced in the heavy mass expansion. Apart from the condition $v \cdot \beta = 0$ (which is indeed satisfied in heavy baryon chiral perturbation theory), no further assumption about the terms entering in (3.1) is made in this section. The discussion will therefore apply to a rather large class of theories of scalars interacting with heavy fermions, not necessarily related to chiral perturbation theory. The further specialization to the pion–nucleon system is reserved until the next chapter.

The action (3.1) is invariant under local gauge transformations

$$\begin{aligned} \xi(x) &\rightarrow R(x)\xi(x), & R(x)^T R(x) &= \mathbf{1}, \\ \eta(x) &\rightarrow U(x)\eta(x), & U(x)^\dagger U(x) &= \mathbf{1}, \\ X_\mu &\rightarrow R \partial_\mu R^{-1} + R X_\mu R^{-1}, \\ Y &\rightarrow R Y R^{-1}, \\ f_\mu &\rightarrow U \partial_\mu U^{-1} + U f_\mu U^{-1}, \\ \alpha &\rightarrow U \alpha R^{-1}, \\ \beta_\mu &\rightarrow U \beta_\mu R^{-1}. \end{aligned} \quad (3.3)$$

Consequently, also the divergent part of the one-loop functional exhibits this symmetry property [10]. The matrix-fields Y , α , β_μ together with their covariant derivatives

$$\begin{aligned} \widehat{\nabla}_\mu Y &:= \partial_\mu Y + [X_\mu, Y], \\ \widehat{\nabla}_\mu \alpha &:= \partial_\mu \alpha + f_\mu \alpha - \alpha X_\mu, \\ \widehat{\nabla}_\mu \beta_\nu &:= \partial_\mu \beta_\nu + f_\mu \beta_\nu - \beta_\nu X_\mu, \end{aligned} \quad (3.4)$$

and the associated “field-strength” tensors

$$\begin{aligned} X_{\mu\nu} &:= \partial_\mu X_\nu - \partial_\nu X_\mu + [X_\mu, X_\nu], \\ f_{\mu\nu} &:= \partial_\mu f_\nu - \partial_\nu f_\mu + [f_\mu, f_\nu] \end{aligned} \quad (3.5)$$

are therefore the appropriate building blocks for the construction of a gauge-invariant action.

The general heat-kernel formalism of the preceding section will now be applied to (3.1). In this case, the matrix-operators A , B , Γ and $\bar{\Gamma}$ defined in (2.9) are given by

$$\begin{aligned} A &= -D^2 - Y, & B &= i v \cdot \mathcal{D}, \\ \Gamma &= \alpha + \beta \cdot D, & \bar{\Gamma} &= \bar{\delta} - \bar{\beta} \cdot \mathcal{D}. \end{aligned} \quad (3.6)$$

As I am considering only terms at most bilinear in the fermionic variables, the form (2.18) for the supermatrix operator is the appropriate one. Employing the method of Ball [11], the relevant diagonal space-time matrix element can be written as

$$\text{str} \left\langle x \left| e^{i\tau K''} \right| x \right\rangle = \text{str} \int d^d k \left\langle x \left| e^{i\tau K''} \right| k \right\rangle \langle k | x \rangle$$

$$\begin{aligned}
&= \text{str} \int \frac{d^d k}{(2\pi)^d} e^{ikx} e^{i\tau K''} e^{-ikx} \\
&= \text{str} \int \frac{d^d k}{(2\pi)^d} e^{i\tau \tilde{K}''} \mathbf{1}, \quad (3.7)
\end{aligned}$$

with

$$\tilde{K}'' = \begin{bmatrix} -D^2 - Y + k^2 + 2ik \cdot D & \sqrt{2\mu} (\bar{\delta} - \bar{\beta} \cdot D + ik \cdot \bar{\beta}) \\ \sqrt{2\mu} (\alpha + \beta \cdot D - ik \cdot \beta) & \mu (iv \cdot D + v \cdot k) \end{bmatrix}. \quad (3.8)$$

The further evaluation of this expression is considerably simplified by the observation that in the following intermediate steps we may restrict ourselves to constant fields [11] X_μ , α , β_μ , f_μ , $Y = -X^2$. As the final result for the one-loop divergences has to be gauge-invariant, no information is lost and the full expression for space-time dependent fields is recovered by the substitutions

$$\begin{aligned}
-X^2 &\rightarrow Y, \\
-[X_\mu, X^2] &\rightarrow \widehat{\nabla}_\mu Y, \\
[X_\mu, X_\nu] &\rightarrow X_{\mu\nu}, \\
f_\mu \alpha - \alpha X_\mu &\rightarrow \widehat{\nabla}_\mu \alpha, \\
f_\mu \beta_\nu - \beta_\nu X_\mu &\rightarrow \widehat{\nabla}_\mu \beta_\nu, \\
[f_\mu, f_\nu] &\rightarrow f_{\mu\nu}. \quad (3.9)
\end{aligned}$$

In this approach, (3.7) reduces to the much simpler expression

$$\text{str} \langle x | e^{i\tau K''} | x \rangle = \int \frac{d^d k}{(2\pi)^d} \text{str} e^{M+N} \quad (3.10)$$

with

$$M = i\tau \begin{bmatrix} k^2 + 2ik \cdot X & 0 \\ 0 & \mu (iv \cdot f + v \cdot k) \end{bmatrix}, \quad (3.11)$$

$$N = i\tau \sqrt{2\mu} \begin{bmatrix} 0 & \bar{\delta} - \bar{\beta} \cdot f + ik \cdot \bar{\beta} \\ \alpha + \beta \cdot X - ik \cdot \beta & 0 \end{bmatrix}.$$

Let us first consider the part bilinear in the fermionic matrix N (generating the terms of the form $\bar{\alpha} \dots \alpha$, $\bar{\alpha} \dots \beta_\mu$, $\bar{\beta}_\mu \dots \alpha$, $\bar{\beta}_\mu \dots \beta_\nu$). The corresponding part of the generating functional (2.10) is just

$$W_{L=1}|_{\bar{\Gamma} \dots \Gamma} := -i \text{Tr} (A^{-1} \bar{\Gamma} B^{-1} \Gamma). \quad (3.12)$$

The appropriate decomposition of the exponential in (3.10) can be performed by using Feynman's "disentangling" theorem [12]:

$$\exp(M + N) = \exp M \text{P}_s \exp \int_0^1 ds \tilde{N}(s) \quad (3.13)$$

with

$$\tilde{N}(s) := e^{-sM} N e^{sM}$$

and

$$\begin{aligned}
\text{P}_s \exp \int_0^1 ds \tilde{N}(s) &:= \sum_{n=0}^{\infty} \int_0^1 ds_1 \int_0^{s_1} ds_2 \\
&\dots \int_0^{s_{n-1}} ds_n \tilde{N}(s_1) \tilde{N}(s_2) \dots \tilde{N}(s_n). \quad (3.14)
\end{aligned}$$

(In the mathematical literature, (3.13) is also known as "Duhamel's formula".) Picking out the part bilinear in N ,

$$\begin{aligned}
\text{str} e^{M+N} &= \int_0^1 ds \int_0^s ds' \text{str} \\
&\times \left[e^{(1-s)M} N e^{(s-s')M} N e^{s'M} \right] + \dots, \quad (3.15)
\end{aligned}$$

a few simple manipulations lead to

$$\begin{aligned}
\text{str} e^{M+N} &= -2\mu\tau^2 \int_0^1 dz e^{i\tau[(1-z)k^2 + z\mu v \cdot k]} \\
&\times \text{tr} \left[(\bar{\delta} - \bar{\beta} \cdot f + i\bar{\beta} \cdot k) e^{-\tau z \mu v \cdot f} \right. \\
&\left. \times (\alpha + \beta \cdot X - i\beta \cdot k) e^{-2\tau(1-z)k \cdot X} \right] + \dots \quad (3.16)
\end{aligned}$$

After integration over z , the μ -dependent terms cancel once the proper-time and the momentum-space integrals are applied. The remaining contribution to $W_{L=1}$ assumes the form

$$\begin{aligned}
W_{L=1}|_{\bar{\Gamma} \dots \Gamma} &= - \int d^d x \int_0^\infty \frac{dt}{t} t^{3-d} \int \frac{d^d l}{(2\pi)^d} e^{iv \cdot l} \\
&\times \text{tr} \left[(\bar{\delta} - \bar{\beta} \cdot f + i\bar{\beta} \cdot l/t) e^{-tv \cdot f} \right. \\
&\left. \times (\alpha + \beta \cdot X - i\beta \cdot l/t) (l^2 + 2itl \cdot X)^{-1} \right], \quad (3.17)
\end{aligned}$$

where a suitable change of the integration variables has been performed. The divergent part (for $d \rightarrow 4$) can now be easily isolated:

$$\begin{aligned}
W_{L=1}^{\text{div}}|_{\bar{\Gamma} \dots \Gamma} &= \Gamma(4-d) \int d^d x \int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{l^2} \text{tr} \left\{ (\bar{\delta} - \bar{\beta} \cdot f) \right. \\
&v \cdot f (\alpha + \beta \cdot X) + \frac{1}{3!} \bar{\beta} \cdot l (v \cdot f)^3 \beta \cdot l \\
&+ \left[(\bar{\delta} - \bar{\beta} \cdot f) (\alpha + \beta \cdot X) + i(\bar{\delta} - \bar{\beta} \cdot f) v \cdot f \beta \cdot l \right. \\
&- i\bar{\beta} \cdot l v \cdot f (\alpha + \beta \cdot X) + \frac{1}{2!} \bar{\beta} \cdot l (v \cdot f)^2 \beta \cdot l \left. \right] \frac{2il \cdot X}{l^2} \\
&+ \left[-i(\bar{\delta} - \bar{\beta} \cdot f) \beta \cdot l + i\bar{\beta} \cdot l (\alpha + \beta \cdot X) \right. \\
&\left. - \bar{\beta} \cdot l v \cdot f \beta \cdot l \right] \frac{4(l \cdot X)^2}{(l^2)^2} - \bar{\beta} \cdot l \beta \cdot l \frac{8i(l \cdot X)^3}{(l^2)^3} \left. \right\}. \quad (3.18)
\end{aligned}$$

The necessary formulas for the l -integration are given in the Appendix. In the last step, one has to identify the appropriate gauge-invariant combinations (constituting a non-trivial check of the calculation) and reconstruct the full result by using (3.9). In this way, I finally obtain:

$$W_{L=1}^{\text{div}}|_{\bar{\Gamma} \dots \Gamma} = \frac{i}{48\pi^2(d-4)} \int d^d x \text{tr} \left\{ -12 \bar{\alpha} v \cdot \widehat{\nabla} \alpha \right.$$

$$\begin{aligned}
& +6 \left[\bar{\alpha} \beta_\mu X^{\mu\nu} v_\nu + \bar{\beta}_\mu \alpha X^{\mu\nu} v_\nu \right] \\
& -3 \left[\bar{\beta} \cdot \beta v \cdot \widehat{\nabla} Y + 2 \bar{\beta}_\mu (v \cdot \widehat{\nabla} \beta^\mu) Y \right] \\
& -4 \bar{\beta}_\mu (v \cdot \widehat{\nabla})^3 \beta^\mu + \bar{\beta} \cdot \beta \widehat{\nabla}_\mu X^{\mu\nu} v_\nu \\
& +6 \bar{\beta}_\mu (v \cdot \widehat{\nabla} \beta_\nu) X^{\mu\nu} + 4 \bar{\beta}_\mu \beta_\nu v \cdot \widehat{\nabla} X^{\mu\nu} \\
& +2 \bar{\beta}_\mu \beta_\nu \widehat{\nabla}^\mu X^{\nu\rho} v_\rho \left. \right\} . \quad (3.19)
\end{aligned}$$

Note that (3.19) has to be real, which is another independent check of the result.

The remaining part of the generating functional with the fermionic operators $\Gamma, \bar{\Gamma}$ turned off,

$$W_{L=1}|_{\Gamma=\bar{\Gamma}=0} = \frac{i}{2} \text{Tr} \ln \frac{A}{A_0} - i \text{Tr} \ln \frac{B}{B_0} , \quad (3.20)$$

does not require any additional effort. A simple calculation (involving a Gaussian momentum-space integration) gives

$$\begin{aligned}
\frac{i}{2} \text{Tr} \ln A|_{\text{div}} &= -\frac{1}{(4\pi)^2(d-4)} \\
&\times \int d^4x \text{tr} \left(\frac{1}{12} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} Y^2 \right) , \quad (3.21)
\end{aligned}$$

which is the standard result obtained by 't Hooft [10] using diagrammatic methods.

The second term in (3.20) vanishes identically, as it corresponds to the closed loop of a “light” fermion component in the heavy mass expansion:

$$\begin{aligned}
\text{Tr} \ln B &= - \int d^d x \int_0^\infty \frac{dt}{t} \int \frac{d^d k}{(2\pi)^d} \text{tr} \left(e^{it(iv \cdot \mathcal{D} + v \cdot k)} \mathbf{1} \right) \\
&= - \int d^d x \int_0^\infty \frac{dt}{t} t^{-d} \int \frac{d^d l}{(2\pi)^d} e^{iv \cdot l} \text{tr} \left(e^{-tv \cdot \mathcal{D}} \mathbf{1} \right) \\
&= 0 ,
\end{aligned}$$

which follows from

$$\int \frac{d^d l}{(2\pi)^d} e^{iv \cdot l} = \delta^{(d)}(v) = 0 .$$

4 Renormalization of the pion–nucleon interaction

The functionals (3.19) and (3.21) are the basic formulas for the analysis of the one-loop divergences to $\mathcal{O}(p^3)$ in heavy baryon chiral perturbation theory. They can be applied to both the two-flavour and the three-flavour case. In the following I shall confine myself to chiral $SU(2)$.

The starting point for the formulation of the effective field theory of the pion–nucleon system is QCD with the two light flavours u, d coupled to external Hermitian fields [13]:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\text{QCD}}^0 + \bar{q} \gamma^\mu \left(\mathcal{V}_\mu + \frac{1}{3} \mathcal{V}_\mu^s + \gamma_5 \mathcal{A}_\mu \right) q - \bar{q} (S - i\gamma_5 P) q , \\
q &= \begin{bmatrix} u \\ d \end{bmatrix} . \quad (4.1)
\end{aligned}$$

$\mathcal{L}_{\text{QCD}}^0$ is the QCD Lagrangian with $m_u = m_d = 0$, S and P are general two-dimensional matrix fields, the isotriplet vector and axial-vector fields $\mathcal{V}_\mu, \mathcal{A}_\mu$ are traceless and the isosinglet vector field \mathcal{V}_μ^s is included to generate the electromagnetic current.

Explicit chiral symmetry breaking is built in by setting $S = \mathcal{M}_{\text{quark}} = \text{diag} [m_u, m_d]$. The chiral group $G = SU(2)_L \times SU(2)_R$ is spontaneously broken to the isospin group $SU(2)_V$. It is realized non-linearly [14] on the Goldstone pion fields ϕ :

$$\begin{aligned}
u_L(\phi) &\xrightarrow{g} g_L u_L(\phi) h(g, \phi)^{-1} , & g &= (g_L, g_R) \in G , \\
u_R(\phi) &\xrightarrow{g} g_R u_R(\phi) h(g, \phi)^{-1} , \quad (4.2)
\end{aligned}$$

where u_L, u_R are elements of the chiral coset space $SU(2)_L \times SU(2)_R / SU(2)_V$ and the compensator field $h(g, \phi)$ is in $SU(2)_V$.

The nucleon doublet Ψ transforms as

$$\Psi = \begin{bmatrix} p \\ n \end{bmatrix} \xrightarrow{g} \Psi' = h(g, \phi) \Psi \quad (4.3)$$

under chiral transformations. The local nature of this transformation requires a connection

$$\Gamma_\mu = \frac{1}{2} \left[u_R^\dagger (\partial_\mu - ir_\mu) u_R + u_L^\dagger (\partial_\mu - i\ell_\mu) u_L \right] \quad (4.4)$$

in the presence of external gauge fields

$$r_\mu = \mathcal{V}_\mu + \mathcal{A}_\mu , \quad \ell_\mu = \mathcal{V}_\mu - \mathcal{A}_\mu \quad (4.5)$$

to define a covariant derivative

$$\nabla_\mu \Psi = (\partial_\mu + \Gamma_\mu - i\mathcal{V}_\mu^s) \Psi . \quad (4.6)$$

To lowest order in the chiral expansion the effective Lagrangian of the pion–nucleon system is [1, 13]

$$\mathcal{L}_{\text{eff}} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle + \bar{\Psi} \left(i \not{\nabla} - m + \frac{g_A}{2} \not{\mathcal{V}} \gamma_5 \right) \Psi , \quad (4.7)$$

with

$$\begin{aligned}
u_\mu &= i \left[u_R^\dagger (\partial_\mu - ir_\mu) u_R - u_L^\dagger (\partial_\mu - i\ell_\mu) u_L \right] , \\
\chi &= 2B(S + iP), \quad \chi_+ = u_R^\dagger \chi u_L + u_L^\dagger \chi^\dagger u_R .
\end{aligned}$$

F, m, g_A are the pion decay constant, the nucleon mass and the neutron decay constant in the chiral limit, whereas B is related to the quark condensate. $\langle \dots \rangle$ stands for the trace in flavour space.

The heavy baryon mass expansion of (4.7) is obtained by introducing velocity-dependent fields

$$\begin{aligned}
N_v(x) &= e^{imv \cdot x} P_v^+ \Psi(x) , & (4.8) \\
H_v(x) &= e^{imv \cdot x} P_v^- \Psi(x) , \\
P_v^\pm &= \frac{1}{2} (1 \pm \not{v}) , \quad v^2 = 1 ,
\end{aligned}$$

leading to

$$\begin{aligned}
\mathcal{L}_{\text{eff}} &= \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle \\
&\quad + \bar{N}_v (iv \cdot \nabla + g_A S \cdot u) N_v + \dots \quad (4.9)
\end{aligned}$$

The additional terms involving the “heavy” fermion components H_v are irrelevant for our present purposes. (For a more detailed discussion the reader is referred to [5, 15].) The only dependence on Dirac matrices in (4.9) is through the spin-vector matrices

$$S^\mu = \frac{i}{2} \gamma_5 \sigma^{\mu\nu} v_\nu, \quad S \cdot v = 0, \quad S^2 = -\frac{3}{4} \mathbf{1}, \quad (4.10)$$

which obey the (anti-) commutation relations

$$\begin{aligned} \{S^\mu, S^\nu\} &= \frac{1}{2} (v^\mu v^\nu - g^{\mu\nu}), \\ [S^\mu, S^\nu] &= i \varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma. \end{aligned} \quad (4.11)$$

To obtain the associated second-order fluctuation Lagrangian $\mathcal{L}_{\text{eff}}^{(2)}$, (4.9) is expanded around the classical fields $\phi_{\text{cl}}, N_{v,\text{cl}}$. In the standard “gauge” $u_R(\phi_{\text{cl}}) = u_L^\dagger(\phi_{\text{cl}}) =: u(\phi_{\text{cl}})$ a convenient choice of the bosonic fluctuation variables ξ_i ($i = 1, 2, 3$) is given by [13]

$$\begin{aligned} u_R(\phi) &= u(\phi_{\text{cl}}) e^{\frac{i \vec{\xi}(\phi) \cdot \vec{\tau}}{2F}}, \quad u_L(\phi) = u^\dagger(\phi_{\text{cl}}) e^{-\frac{i \vec{\xi}(\phi) \cdot \vec{\tau}}{2F}}, \\ \vec{\xi}(\phi_{\text{cl}}) &= 0, \end{aligned} \quad (4.12)$$

where $\vec{\tau}$ denotes the Pauli matrices. For the fermion fields I write $N_v = N_{v,\text{cl}} + \eta$. In this way I get

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{(2)} &= \frac{1}{2} \left[(d_{\mu k i} \xi_i)(d_{k j}^\mu \xi_j) - \sigma_{ij} \xi_i \xi_j \right] + \frac{1}{8F^2} \bar{N}_v \\ &\quad \times \left[i \xi_i [\tau_i, \tau_k] (v \cdot d_{k j} \xi_j) + g_A \xi_i [\tau_i, [S \cdot u, \tau_j]] \xi_j \right] N_v \\ &\quad + \frac{1}{F} \bar{N}_v \left[\frac{i}{4} [v \cdot u, \tau_i] \xi_i - g_A S_\mu \tau_i (d_{ij}^\mu \xi_j) \right] \eta \\ &\quad + \frac{1}{F} \bar{\eta} \left[\frac{i}{4} [v \cdot u, \tau_i] \xi_i - g_A S_\mu \tau_i (d_{ij}^\mu \xi_j) \right] N_v \\ &\quad + \bar{\eta} (i v \cdot \nabla + g_A S \cdot u) \eta, \end{aligned} \quad (4.13)$$

where

$$\begin{aligned} d_{ij}^\mu &= \delta_{ij} \partial^\mu + \gamma_{ij}^\mu, \quad \gamma_{ij}^\mu = -\frac{1}{2} \langle \Gamma^\mu [\tau_i, \tau_j] \rangle, \\ \sigma_{ij} &= \frac{1}{4} \langle (u \cdot u + \chi_+) \delta_{ij} - \tau_i u_\mu \tau_j u^\mu \rangle. \end{aligned} \quad (4.14)$$

Note that the quantities $N_v, u_\mu, \Gamma_\mu, \chi_+$ in (4.13) are to be taken at the solutions of the classical equations of motion. (The subscript “cl” has only been dropped for simplicity.)

It is now easy to verify that the action associated with (4.13) can indeed be written in the standard form (3.1) by setting

$$\begin{aligned} X_\mu &= \gamma_\mu + g_\mu, \quad g_{ij}^\mu = -\frac{i v^\mu}{8F^2} \bar{N}_v [\tau_i, \tau_j] N_v, \\ i &= 1, 2, 3, \quad Y = \sigma + s, \\ s_{ij} &= \frac{g_A}{4F^2} \bar{N}_v (2 \delta_{ij} S \cdot u - \tau_i S \cdot u \tau_j - \tau_j S \cdot u \tau_i) N_v, \\ f_\mu &= \Gamma_\mu - i \mathcal{V}_\mu^s - i v_\mu g_A S \cdot u \\ \alpha_{ai} &= \frac{i}{4F} ([v \cdot u, \tau_i] N_v)_a, \quad a = 1, 2, \\ (\beta_\mu)_{ai} &= -\frac{g_A}{F} S_\mu (\tau_i N_v)_a. \end{aligned} \quad (4.15)$$

Let us first consider the one-loop divergences generated by (3.21). Using (4.15),

$$\text{tr } Y^2 = \text{tr } \sigma^2 + 2 \text{tr } (\sigma s) + \dots \quad (4.16)$$

and

$$\begin{aligned} \text{tr } (X_{\mu\nu} X^{\mu\nu}) &= \text{tr } \gamma_{\mu\nu} \gamma^{\mu\nu} + 4 \text{tr } \{ \gamma_{\mu\nu} (\partial^\mu g^\nu + [\gamma^\mu, g^\nu]) \} \\ &\quad + \dots, \end{aligned} \quad (4.17)$$

where

$$\gamma_{\mu\nu} := \partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu + [\gamma_\mu, \gamma_\nu]. \quad (4.18)$$

The first terms on the right-hand sides of (4.16) and (4.17) are purely mesonic; they determine the divergence structure of the well-known Gasser–Leutwyler functional of $\mathcal{O}(p^4)$ [13]. The second ones are bilinear in the fermion fields, whereas the dots refer to irrelevant terms $\sim (\bar{N}_v \dots N_v)^2$. To facilitate the comparison with [5], I write the fermion bilinears extracted from (3.21) in the following form:

$$\begin{aligned} W_{L=1}^{\text{div}}|_{\Gamma=\bar{\Gamma}=0} &= \int d^4x \bar{N}_v \Sigma_1^{\text{div}} N_v, \\ \Sigma_1^{\text{div}} &= -\frac{1}{8\pi^2 F^2 (d-4)} \hat{\Sigma}_1. \end{aligned} \quad (4.19)$$

For $\hat{\Sigma}_1$ I find

$$\begin{aligned} \hat{\Sigma}_1 &= -\frac{i}{6} (\nabla^\mu \Gamma_{\mu\nu} v^\nu) + \frac{g_A}{4} (\langle u \cdot u + \chi_+ \rangle S \cdot u \\ &\quad + \langle S \cdot u u_\mu \rangle u^\mu), \end{aligned} \quad (4.20)$$

where

$$\Gamma_{\mu\nu} := \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu]. \quad (4.21)$$

This result agrees with the corresponding expression in (36) of [5]. Note that I have used several $SU(2)$ relations to arrive at a simpler form for $\hat{\Sigma}_1$ in (4.20).

The one-loop divergences originating from (3.19) are again presented in the form

$$\begin{aligned} W_{L=1}^{\text{div}}|_{\Gamma \dots \bar{\Gamma}} &= \int d^4x \bar{N}_v \Sigma_2^{\text{div}} N_v, \\ \Sigma_2^{\text{div}} &= -\frac{1}{8\pi^2 F^2 (d-4)} \hat{\Sigma}_2. \end{aligned} \quad (4.22)$$

Inserting (4.15) in (3.19), I obtain:

$$\begin{aligned} \hat{\Sigma}_2 &= i \left\{ \frac{1}{4} [2(v \cdot u)^2 + \langle (v \cdot u)^2 \rangle] v \cdot \nabla + \frac{1}{2} v \cdot u (v \cdot \nabla v \cdot u) \right. \\ &\quad + \frac{1}{4} \langle v \cdot u (v \cdot \nabla v \cdot u) \rangle \left. \right\} + g_A \left\{ -\frac{1}{2} v \cdot u \langle S \cdot u v \cdot u \rangle \right. \\ &\quad + \frac{1}{4} \langle S \cdot u (v \cdot u)^2 \rangle - S^\mu v^\nu [\Gamma_{\mu\nu}, v \cdot u] \left. \right\} \\ &\quad + i g_A^2 \left\{ -\frac{3}{2} (v \cdot \nabla)^3 - \frac{5}{6} (\nabla^\mu \Gamma_{\mu\nu} v^\nu) \right. \\ &\quad + i \varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma [2\Gamma_{\mu\nu} v \cdot \nabla + (v \cdot \nabla \Gamma_{\mu\nu})] \\ &\quad \left. - \frac{3}{32} (v \cdot \partial \langle 4u \cdot u + 3\chi_+ \rangle) - \frac{3}{16} \langle 4u \cdot u + 3\chi_+ \rangle v \cdot \nabla \right\} \end{aligned}$$

$$\begin{aligned}
& +g_A^3 \left\{ -\frac{1}{2} S \cdot u (v \cdot \nabla)^2 - 2S^\mu \langle S \cdot u \Gamma_{\mu\nu} \rangle S^\nu \right. \\
& -\frac{1}{2} (v \cdot \nabla S \cdot u) v \cdot \nabla - \frac{1}{6} ((v \cdot \nabla)^2 S \cdot u) \\
& \left. -\frac{1}{4} u_\mu \langle u^\mu S \cdot u \rangle - \frac{1}{16} S \cdot u \langle \chi_+ \rangle \right\} \\
& +ig_A^4 S_\mu \left\{ [2(S \cdot u)^2 - 4 \langle (S \cdot u)^2 \rangle] v \cdot \nabla \right. \\
& +\frac{2}{3} (v \cdot \nabla S \cdot u) S \cdot u \\
& \left. +\frac{4}{3} S \cdot u (v \cdot \nabla S \cdot u) - 4 \langle S \cdot u (v \cdot \nabla S \cdot u) \rangle \right\} S^\mu \\
& +g_A^5 S_\mu \left\{ \frac{2}{3} (S \cdot u)^3 - \frac{4}{3} \langle (S \cdot u)^3 \rangle \right\} S^\mu, \quad (4.23)
\end{aligned}$$

which is in agreement with the corresponding result in [5]. (Note that “+–” in the fourth line of (53) in [5] should be read as a minus sign.)

5 Conclusions

I have shown that the super-heat-kernel technique constitutes the appropriate theoretical tool for analyzing the one-loop divergences in systems with (non-renormalizable) boson–fermion interactions. I recall here the essential ingredients that were combined to arrive at an efficient computational scheme:

- The one-loop functional is written in terms of the superdeterminant of a suitably chosen supermatrix operator.
- The associated super-heat-kernel representation is the appropriate form of the one-loop functional for studying its divergence structure.
- It is easier to determine the diagonal heat-kernel matrix elements directly by inserting a complete set of plane waves instead of calculating the Seeley–DeWitt coefficients with two different space-time arguments and taking the coincidence-limit at the end.
- The heat-kernel-representation is perfectly well defined also for supermatrices with first-order (fermion) differential operators¹.
- The second-order fluctuation action is invariant under a local gauge transformation. As a consequence, this symmetry property is also shared by the divergence functional.
- At intermediate stages, the calculation can be carried out with constant (classical) fields, avoiding cumbersome manipulations with derivatives acting on space-time dependent objects. At the end, the general result is recovered by gauge invariance.
- Feynman’s disentangling theorem allows the proper decomposition of the exponential of a sum of non-commuting terms.

¹ Note, however, that “squaring” of the fermionic differential operator may simplify the analysis in theories where the full relativistic Dirac operator is still present [7]

- With the divergence functional given in compact form, the one-loop renormalization of effective quantum field theories becomes an easy task, requiring only a few purely algebraic operations.

The application to heavy baryon chiral perturbation theory with two flavours served as an explicit example. A previous result for the counterterms to $\mathcal{O}(p^3)$ was confirmed.

With the super-heat-kernel method at hand, the systematic study of effective field theories at the one-loop level is simplified considerably. I am giving here a small selection of possible applications and extensions of the present work:

- The treatment of the meson–baryon interaction with three flavours is completely analogous to the two-flavour case discussed before.
- The inclusion of fields with higher spin (photon, Δ -resonance, etc.) is straightforward. Their components are simply added to the bosonic and fermionic sectors, respectively.
- The completion of the one-loop renormalization for the pion–nucleon interaction up to $\mathcal{O}(p^4)$ may be achieved by a suitable extension of (3.1).
- For the analysis of fermionic bound states, the complete form (2.16) of the supermatrix operator must be used, as terms quartic in the fermion fields are relevant in this case.
- In analogy to the mesonic case [13], the super-heat-kernel representation might also be useful for the finite part of the one-loop functional with two external baryons.

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Appendix

I consider first the integrals

$$I_n(v^2) := \int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{(l^2 + i\epsilon)^n} = f_n(d)(v^2)^{\frac{2n-d}{2}} \quad (\text{A.1})$$

with an arbitrary four-vector v_μ . The $f_n(d)$ are given by

$$f_1(d) = (-i)^{d-1} \frac{\Gamma(d-2)}{(4\pi)^{\frac{d-1}{2}} \Gamma(\frac{d-1}{2})} \quad (\text{A.2})$$

and

$$f_n(d) = \frac{1}{2^{n-1} (n-1)! (d-2n) \dots (d-4)} f_1(d), \quad n = 2, 3, \dots \quad (\text{A.3})$$

The momentum-space integrals occurring in (3.19) are now obtained by differentiating (A.1) a sufficient number

of times with respect to v_μ and setting $v^2 = 1$ at the end:

$$\int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{l^2} \xrightarrow{d \rightarrow 4} \frac{i}{4\pi^2}, \quad (\text{A.4})$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{l^2} l_\mu l_\nu \xrightarrow{d \rightarrow 4} \frac{i}{2\pi^2} (g_{\mu\nu} - 4v_\mu v_\nu), \quad (\text{A.5})$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{(l^2)^2} l_\mu \xrightarrow{d \rightarrow 4} -\frac{v_\mu}{8\pi^2}, \quad (\text{A.6})$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{(l^2)^2} l_\mu l_\nu \xrightarrow{d \rightarrow 4} \frac{i}{8\pi^2} (g_{\mu\nu} - 2v_\mu v_\nu), \quad (\text{A.7})$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{(l^2)^2} l_\mu l_\nu l_\rho \xrightarrow{d \rightarrow 4} \frac{1}{4\pi^2} [-(g_{\mu\nu} v_\rho + \dots) + 4v_\mu v_\nu v_\rho], \quad (\text{A.8})$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{(l^2)^3} l_\mu l_\nu l_\rho \xrightarrow{d \rightarrow 4} \frac{1}{32\pi^2} [-(g_{\mu\nu} v_\rho + \dots) + 2v_\mu v_\nu v_\rho], \quad (\text{A.9})$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{(l^2)^3} l_\mu l_\nu l_\rho l_\sigma \xrightarrow{d \rightarrow 4} \frac{i}{32\pi^2} [(g_{\mu\nu} g_{\rho\sigma} + \dots) - 2(g_{\mu\nu} v_\rho v_\sigma + \dots) + 8v_\mu v_\nu v_\rho v_\sigma], \quad (\text{A.10})$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{e^{iv \cdot l}}{(l^2)^4} l_\mu l_\nu l_\rho l_\sigma l_\tau \xrightarrow{d \rightarrow 4} \frac{1}{192\pi^2} [-(g_{\mu\nu} g_{\rho\sigma} v_\tau + \dots) + 2(g_{\mu\nu} v_\rho v_\sigma v_\tau + \dots) - 8v_\mu v_\nu v_\rho v_\sigma v_\tau]. \quad (\text{A.11})$$

The dots indicate the necessary symmetrizations.

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